Find the values of the constants a and b if the straight lines

1. $ax + 5y = 3$, and $4x + by = 1$ intersect at the point (2, -1)

ANSWER: a = 4, b = 7

[2a – 5 = 3, 2a = 8, a = 4, 8 – b = 1, b = 7]

2. $3x + ay = 4$, and $bx - 5y = 6$ intersect at the point (2, -2)

ANSWER: a = 1, b = -2

[6 – 2a = 4, 2a = 2, a = 1, 2b + 10 = 6, 2b = -4, b = --2]

3. $ax + 2y = -3$, and $2x + by = 6$ meet at the point (1, 2)

ANSWER: a = -7, b = 2

[a + 4 = -3, a = -7, 2 + 2b = 6, 2b = 4, b = 2]

Solve for x if the determinant of the matrix A has the given value.

1. $A = \begin{pmatrix} 2x & x \\ 3 & x \end{pmatrix}$, $\det A = 5$

ANSWER: x = 5/2, -1

[$2x^2 - 3x - 5 = 0$, $2x^2 - 5x + 2x - 5 = (x + 1)(2x - 5) = 0$, x = 5/2, - 1]

2. $A = \begin{pmatrix} 3x & 1 \\ x & 2x \end{pmatrix}$, $\det A = 7$

ANSWER: x = 7/6, -1

[$6x^2 - x - 7 = 0$, $6x^2 - 7x + 6x - 7 = (x + 1)(6x - 7) = 0$, x = 7/6, -1]

3. $A = \begin{pmatrix} 3x & 2 \\ x & 5x \end{pmatrix}$, $\det A = 1$

ANSWER: x = 1/3, -1/5

[$15x^2 - 2x - 1 = (3x - 1)(5x + 1) = 0$, x = 1/3, x = -1/5]

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1. Factorize completely $a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

ANSWER: $(a + b)^4$

2. In how many ways can 5 persons be seated in a row of 5 seats.

ANSWER: 120

[5! = 5 x 4 x 3 x 2 x 1 = 20 x 6 = 120]

3. Find the coordinates of the point of inflexion of the curve $y = x^3 - 6x^2 + 16x$.

ANSWER: (2, 16)

[$dy/dx = 3x^2 - 12x + 16$, $d^2y/dx^2 = 6x - 12 = 0$, $x = 2$, $y = 8 - 24 + 32 = 16$, (2, 16)]

Find the degree measures of the interior angles of a triangle if

1. the exterior angles are in the ratio 3: 4 : 5

ANSWER: 90°, 60°, 30°

[$3x + 4x + 5x = 12x = 360$, $x = 30$, exterior angles are 90, 120, 150, interior angles are $180 - 90 = 90$, $180 - 120 = 60$, $180 - 150 = 30$]

2. the exterior angles are in the ratio 11: 12: 13

ANSWER: 70°, 60°, 50°

[$11x + 12x + 13x = 36x = 360$, $x = 10$, exterior angles are 110, 120, 130, interior angles are $180 - 110 = 70$, $180 - 120 = 60$, $180 - 130 = 50$]

3. the exterior angles are in the ratio 5 : 6 : 7

ANSWER: 80°, 60°, 40°

[$5x + 6x + 7x = 18x = 360$, $x = 20$, exterior angles are 100, 120, 140, interior angles are $180 - 100 = 80$, $180 - 120 = 60$, $180 - 140 = 40$]

Find the values of A, B, C such that

1. $9x^2 + 12x + A = (3x + B)^2$

ANSWER: A = 4, B = 2

[$9x^2 + 12x + A = 9x^2 + 6Bx + B^2$, $12 = 6B$, $B = 2$, $A = B^2 = 2^2 = 4$]

2. $4x^2 + 16x + 25 = A(x + B)^2 + C$

ANSWER: A = 4, B = 2, C = 9

[$4x^2 + 16x + 25 = 4(x^2 + 4x) + 25 = 4(x + 2)^2 + 25 - 16$, $A = 4$, $B = 2$, $C = 9$]

3. $5x^2 - 30x - 6 = A(x + B)^2 + C$

ANSWER: A = 5, B = -3, C = -51

[$5(x^2 - 6x) - 6 = 5(x - 3)^2 - 45 - 6 = A(x + B)^2 + C$, $A = 5$, $B = -3$, $C = - 51$]

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1. Find the sum to infinity of the series 5 - 5/3 + 5/9 - 5/27 + . . .

ANSWER: 15/4, or 3.75

[exponential series $a = 5$, $r = -1/3$, $S_\infty = a/(1 - r) = 5/(1 + 1/3) = 15/4$]

2. Find the solution set of the inequality $2x^2 - 3x - 5 > 0$.

ANSWER: {x: x > 5/2, or x < - 1}
[$2x^2 - 3x - 5 = (2x - 5)(x + 1) > 0$, $x > 5/2$ or $x < -1$]

3. Find the sum in radians of the interior angles of a polygon of 17 sides.

ANSWER: 15π radians
[$(n - 2)\pi = (17 - 2)\pi = 15\pi$ radians]

Solve the equation for x from the logarithmic equation

1. $\log_6 x + \log_6 x^2 = 3$

ANSWER: x = 6
[$\log_6 x + 2\log_6 x = 3\log_6 x = 3$, $\log_6 x = 1$, $x = 6$]

2. $\log_3 x - \log_3(x - 1) = 2$

ANSWER: x = 9/8
[$\log_3(x/(x - 1)) = 2$, $x/(x - 1) = 9$, $x = 9(x - 1)$, $8x = 9$, $x = 9/8$]

3. $\log_2 x = \log_2 (x + 3) - 1$

ANSWER: x = 3
[$\log_2 (x/(x + 3)) = \log_2(1/2)$, $x/(x + 3) = ½$, $2x = x + 3$, $x = 3$]

Find the equation of the locus of the point P (x, y) moving in the coordinate plane such that AP = BP given

1. A(-4, 2) and B(2, - 4)

ANSWER: y = x

[$(x + 4)^2 + (y - 2)^2 = (x - 2)^2 + (y + 4)^2$, $8x - 4y = -4x + 8y$, $y = x$]

2. A(3, -2) and B(2, - 3)

ANSWER: y = -x
[$(x - 3)^2 + (y + 2)^2 = (x - 2)^2 + (y + 3)^2$, $-6x + 4y = -4x + 6y$, $-2x = 2y$, $y = -x$]

3. A(2, 4) and B (-2, -4)

ANSWER: y = - x/2
[$(x - 2)^2 + (y - 4)^2 = (x + 2)^2 + (y + 4)^2$, $-4x - 8y = 4x + 8y$, $16y = -8x$,
$y = -x/2$]

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1. Find the equation of the tangent to the curve $y^2 = 4x$ at the point A(1, -2)

ANSWER: y = -x – 1, or x + y + 1 = 0
[2ydy/dx = 4, dy/dx = 2/y, m = 2/-2 = -1, y + 2 = -1(x – 1), y = - x - 1]

2. Solve for x given $(1/25)^{x+2} = 125^{x-2}$

ANSWER: x = 2/5
[$5^{-2(x+2)} = 5^{3(x-2)}$, -2x – 4 = 3x – 6, 5x = 2, x = 2/5]

3. If $(2x + 3)/(x^2 – x - 6) = A/(x – 3) + B(x + 2)$, find the value of (A + B)

ANSWER: 2
[2x + 3 = A(x + 2) + B(x – 3), for x = 3, 9 = 5A, A = 9/5, for x = -2, -1 = -5B, B = 1/5
A + B = 9/5 + 1/5 = 2]

Find the coordinates of the vertices of a triangle whose sides are along the lines

1. x + y = 3, x = 4, y = 5

ANSWER: (4, 5), (4, -1), (-2, 5)
[x + y = 3 and x = 4, y = - 1, (4, -1), x + y = 3 and y = 5, x = -2, (-2, 5),
x = 4 and y = 5 gives (4, 5)]

2. x – y = 5, x = -2, y = 3

ANSWER: (8, 3), (-2, 3), (-2, -7)

[x – y = 5 and x = -2, y = -7, (-2, -7), x – y = 5 and y = 3, x = 8, (8, 3),
x = -2 and y = 3 gives (-2, 3)]

3. 2x + y = 8, x = 3, y = 4

ANSWER: (3, 2), (2, 4), (3, 4)

[2x + y = 8 and x = 3, y = 2, (3, 2), 2x + y = 8 and y = 4, gives x = 2, (2, 4),
x = 3, y = 4 gives (3, 4)]

Given that A and B are acute angles and sinA = 3/5, cosB = 1/√2, evaluate.

1. sin(A – B)

ANSWER: -1/5√2, or - √2/10

[sin(A – B) = sinAcosB – cosA sinB = (3/5)(1/√2) – (4/5)(1/√2) =
-1/5√2 = -√2/10]

2. $\cos(A - B)$

ANSWER: $7/5\sqrt{2}$ or $7\sqrt{2}/10$

[$\cos(A - B) = \cos A\cos B + \sin A\sin B = (4/5)(1/\sqrt{2}) + (3/5)(1/\sqrt{2})$
$= 7/5\sqrt{2} = 7\sqrt{2}/10$]

3. $\sin(A + B)$

ANSWER: $7/5\sqrt{2}$ or $7\sqrt{2}/10$

[$\sin(A + B) = \sin A\cos B + \cos A\sin B = (3/5)(1/\sqrt{2}) + (4/5)(1/\sqrt{2}) =$
$7/5\sqrt{2} = 7\sqrt{2}/10$]

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1. Given $x = \cos\theta$, express $x/\sqrt{(1 - x^2)}$ as a trigonometric ratio.

ANSWER: $\cot\theta$, or $1/\tan\theta$
[$\cos\theta/\sqrt{(1 - \cos^2\theta)} = \cos\theta/\sqrt{\sin^2\theta} = \cos\theta/\sin\theta = \cot\theta$]

2. Find the coordinates of the point of inflexion of the curve $y = 2x^3 - 6x^2 + 5x - 2$.

ANSWER: (1, -1)
[$dy/dx = 6x^2 - 12x + 5$, $d^2y/dx^2 = 12x - 12 = 0$, $x = 1$, $y = 2 - 6 + 5 - 2 = -1$, (1, -1)]

3. If M(a, - 2) is the midpoint of the line segment joining the points A(6, -4) and

B(a, b) find the coordinates of B.
ANSWER: (6, 0)
[$(a, -2) = ((a + 6)/2, (b - 4)/2)$, $a = (a + 6)/2$, $a = 6$, $-2 = (b - 4)/2$, $b = 0$, B(6, 0)]

Find the equation of the image of the given curve

1. $(x - 3)^2 + (y + 3)^2 = 10$, after a reflection in the x- axis,

ANSWER: $(x - 3)^2 + (y - 3)^2 = 10$ accept $(x - 3)^2 + (3 - y)^2 = 10$
[$(x, y) \to (x, -y)$, $(x - 3)^2 + (-y + 3)^2 = 10$, $(x - 3)^2 + (y - 3)^2 = 10$]

2. $y^2 = 4x$, after a reflection in the line $y = x$,

ANSWER: $x^2 = 4y$ or $y = x^2/4$
[$(x, y) \to (y, x)$, $x^2 = 4y$, or $y = x^2/4$]

3. $x^3 - y^3 = 10$, after a reflection in the y-axis.

ANSWER: $x^3 + y^3 = -10$
[$(x, y) \to (-x, y)$, $(-x)^3 - y^3 = -x^3 - y^3 = 10$, $x^3 + y^3 = -10$]

Find the acceleration vector of a particle of mass m acted upon by the forces

1. (**3i - 4j**)N, (-**5i + j**)N, (**5i – 3j**)N and m = 0.5 kg,

ANSWER: (6i – 12j) m/s² [(3i - 4j) + (-5i + j) + (5i – 3j) = (3i – 6j) = 0.5a, a = 2(3i – 6j) = (6i – 12j)m/s²]

2. (**4i – 2j**)N, (**2i + 3j**)N, (- **3i + 4j**)N and m = 0.2 kg

ANSWER: (15i + 25j) m/s² [(4i – 2j) + (2i + 3j) + (-3i + 4j) = (3i + 5j) = 0.2a, a = 5(3i + 5j) = 15i + 25j m/s²]

3. (**5i – 7j**) N, (**3i + j**) N + (**i – 3j**) N, and m = 0.3 kg

ANSWER: (30i – 30j) m/s² [(5i – 7j) + (3i + j) + (i – 3j) = (9i – 9j) = 0.3a, a = 10(9i – 9j)/3 = (30i – 30j) m/s²]

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1. Find the solution set of the equation | x |= -x

(Read as 'absolute value of x = -x ')
ANSWER: {x: x ≤ 0}

2. A committee of 3 is to be formed from 3 men and 3 women. In how ways can this be done if there are 2 women and 1 man on the committee

ANSWER: 9
[$3C_2$ x $3C_1$ = 3 x 3 = 9]

3. Describe the set of points (x, y) such that $4 < x^2 + y^2 < 9$

ANSWER: Region between 2 concentric circles with center at the origin and having radii 2 and 3.

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u and **v** are two non-zero vectors and θ is the angle between them. What can you deduce about θ given that

1. **u.v** = 0 (scalar product of u and v is zero)

ANSWER: θ is a right angle, or θ = 90°.

2. **u.v** > 0 (scalar product of u and v is positive)

ANSWER: θ is an acute angle.

3. **u.v** < 0 (scalar product of u and v is negative)

ANSWER: θ is an obtuse angle.

Solve the logarithmic equation for real x.

1. $\log_2 x + \log_2(x + 2) = \log_2(x + 6)$

ANSWER: x = 2
[$x(x + 2) = x + 6$, $x^2 + x - 6 = (x + 3)(x - 2) = 0$, $x = 2$]

2. $\log_3 x + \log_3(x - 8) = 2$

ANSWER: x = 9
[$x(x - 8) = 3^2$, $x^2 - 8x = 9$, $x^2 - 8x - 9 = (x - 9)(x + 1) = 0$, $x = 9$]

3. $\log x + \log(x - 3) = 1$

ANSWER: x = 5
[$x(x - 3)= 10$, $x^2 - 3x - 10 = (x - 5)(x + 2) = 0$, $x = 5$]

Evaluate the given limit

1. $\lim\limits_{x \to -1} \frac{(3x^2 + 4x + 1)}{(x + 1)}$

ANSWER: -2

[$(3x^2 + 4x + 1)/(x + 1) = (3x + 1)(x + 1)/(x + 1) = 3x + 1$, Limit = $-3 + 1$

= -2]

2. $\lim\limits_{x \to 1} \frac{(3x^2 - 2x - 1)}{(x - 1)}$

ANSWER: 4

[$(3x^2 - 2x - 1)/(x - 1) = (3x + 1)(x - 1)/(x - 1) = (3x + 1)$, Limit = $3 + 1 = 4$]

3. $\lim\limits_{x \to 3} \frac{(2x^2 - 5x - 3)}{(x - 3)}$

ANSWER: 7
[$(2x^2 - 5x - 3)/(x - 3) = (2x + 1)(x - 3)/(x - 3) = 2x + 1$, Limit = $2(3) + 1 = 7$]

Find the equation of the line passing through the point A(2, -2) and which is

1. perpendicular to the line through the points B(-1, 2) and C(2, 1)

ANSWER: y = 3x - 8
[$m_{bc} = -1/3$, perpendicular line $m = 3$, $y + 2 = 3(x - 2)$, $y = 3x - 8$]

2. parallel to the line through the points B(3, 1) and C(1, 3)

ANSWER: y = -x

[m_{bc} = 2/-2 = -1, parallel line m = -1, (y + 2) = -1(x – 2), y = -x]

3. perpendicular to the line through the points B(2, 3) and C(- 3, -2)

ANSWER: y = -x
[m_{BC} = -5/-5 = 1, perpendicular line m = -1, (y - 2) = -1(x + 2), y = -x]

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1. Solve the equation $x^4 = 81x^2$

ANSWER: x = 0, ± 9
[$x^2(x^2 - 81) = 0$, x = 0 or x = ± 9]

2. Find the equation of the line making intercepts of -3 on the x-axis and 2 on the y-axis.

ANSWER: **x/-3 + y/2 = 1**, or **-2x + 3y = 6**, or **y = (2/3)x + 2**
[x/-3 + y/2 = 1, -2x + 3y = 6, or y = (2/3)x + 2]

3. Find the set of values of x for which the function $y = x^3 - 2x^2 + x - 2$ is increasing.

ANSWER: {x: x > 1 or x < 1/3}
[$dy/dx = 3x^2 - 4x + 1 = (3x - 1)(x - 1) > 0$, x > 1 or x < 1/3]

A linear transformation is given by T: (x, y) → (2x + y, 5x + 3y). Find the coordinates of the point A(x, y) if its image under the transformation is

1. (1, 1)

ANSWER: (2, -3) (Accept x = 2, y = -3)
[2x + y = 1, 5x + 3y = 1, 3(2x + y) – (5x + 3y) = 3 – 1 = 2, x = 2, y = -3]

2. (3, 2)

ANSWER: (7, -11) (Accept x = 7, y = -11)
[2x + y = 3, 5x + 3y = 2, 3(2x + y) – (5x + 3y) = x = 9 – 2 = 7, y = -11]

3. (-2, 3)

ANSWER: (-9, 16) (Accept x = -9, y = 16)
[2x + y = -2, 5x + 3y = 3, 3(2x + y) – (5x + 3y) = 3(-2) – 3, x = -9, y = 16]

Find the equation of the tangent to the curve

1. $y^2 = 4(x + 2)$ at the point A(2, 4)

ANSWER: y = ½x + 3

[$2y(dy/dx) = 4$, $dy/dx = 2/y$, $m = ½$, $y – 4 = ½ (x – 2)$, $y = ½x + 3$]

2. $y^2 = 8(x – 4)$ at A(6, 4)

ANSWER: $y = x - 2$
[$2y(dy/dx) = 8$, $dy/dx = 4/y$, $m = 1$, $y – 4 = x – 6$, $y = x – 2$]

3. $y^2 = - 4(x – 2)$ at (-2, -4)

ANSWER: $y = ½x - 3$
[$2y(dy/dx) =– 4$, $dy/dx = -2/y$, $m = ½$, $y + 4 = ½ (x + 2)$, $y = ½x – 3$]

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1. Find the equation of the locus of the point P(x, y) given that the vectors

a = (x + 2)**i** – 4j and b = 4**i** + (y – 2)**j** are perpendicular.
ANSWER: x – y + 4 = 0, or **y = x + 4**
[a.b = (x + 2)4 – 4(y – 2) = 0, 4x + 8 - 4y + 8 = 0, x – y + 4 = 0, or y = x + 4]

2. Simplify ((y/x) – (x/y))/((1/y) – (1/x))

ANSWER: -(y + x)
[$(y^2 – x^2)/(x – y) = (y – x)(y + x)/(x – y) = -(y + x)$]

3. Rationalize the denominator of $\sqrt{10}/(\sqrt{5} – 2)$ and simplify.

ANSWER: $5\sqrt{2} + 2\sqrt{10}$
[$\sqrt{10}(\sqrt{5} + 2)/(5 – 4) = 5\sqrt{2} + 2\sqrt{10}$]

Solve the equation for x

1. $\log(x^2 – 15x) = 2$

ANSWER: x = 20, -5
[$x^2 – 15x = 100$, $x^2 – 15x – 100 = (x – 20)(x + 5) = 0$, x = 20, -5]

2. $\log_3(x^2 + 6x) = 3$

ANSWER: x = 3, -9
[$x^2 + 6x = 27$, $x^2 + 6x – 27 = (x + 9)(x – 3) = 0$, x = 3, -9]

3. $\log_5(x^2 + 24x) = 2$,

ANSWER: x = -25, 1
[$x^2 + 24x = 25$, $x^2 + 24x – 25 = (x + 25)(x - 1) = 0$, x = 1, -25]

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n is a positive integer. Solve for n from the equation

1. $nC_2 = 36$ (read as 'n combination 2' = 36)

ANSWER: n = 9
[$n(n - 1)/2 = 36$, $n^2 - n - 72 = (n - 9)(n + 8) = 0$, $n = 9$]

2. $(n + 1)C_2 = 78$

ANSWER: n = 12
[$(n + 1)n/2 = (78)$, $n^2 + n - 156 = (n + 13)(n - 12) = 0$, $n = 12$]

3. $(n - 1)C_2 = 45$

ANSWER: n = 11
[$(n - 1)(n - 2)/2 = 45$, $n^2 - 3n + 2 = 90$, $n^2 - 3n - 88 = (n - 11)(n + 8) = 0$, $n = 11$]

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1. A stone is projected vertically up with velocity 30 m/s. Find the maximum height reached from the point of projection. (Take $g = 10\ m/s^2$)

ANSWER: 45 m
[$v = u - gt = 30 - 10t = 0$, $t = 3$, $s = ut - ½ gt^2 = 30(3) - ½ (10)3^2 = 90 - 45 = 45$ m]

2. Express the vector **a = -4i – 4j** as a bearing

ANSWER: (4√2, 225°)
[|**a**| = $\sqrt{(16 + 16)} = 4\sqrt{2}$, $\tan\theta = -1$ in 3rd quadrant, $\theta = 225°$, $(4\sqrt{2}, 225°)$]

3. Find the coordinates of the center of the circle $3x^2 + 3y^2 + 7x + 5y = 15$

ANSWER: (-7/6, - 5/6)
[$x^2 + y^2 + (7/3)x + (5/3)y = 5$, hence center is (-7/6, -5/6)]

Find the coordinates of the vertex of the given quadratic curve.

1. $y = x^2 - 6x + 7$

ANSWER: (3, -2)
[$x^2 - 6x + 9 + (7 - 9) = (x - 3)^2 - 2$, vertex = (3, -2)]

2. $y = x^2 + 8x - 3$

ANSWER: (-4, -19)
[$x^2 + 8x + 16 + (- 3 - 16) = (x + 4)^2 -19$, vertex = (-4, -19)]

3. $y = 2x^2 + 4x - 5$

ANSWER: (-1, -7)
[$2(x^2 + 2x + 1) - 5 - 2 = 2(x + 1)^2 - 7$, vertex = (-1, -7)]

Find the value of n given that

1. $\ldots 123_n = 38_{10}$

ANSWER: n = 5
[$n^2 + 2n + 3 = 38$, $n^2 + 2n - 35 = (n + 7)(n - 5) = 0$, $n = 5$]

2. $132_n = 72_{10}$

ANSWER: n = 7
[$n^2 + 3n + 2 = 72$, $n^2 + 3n - 70 = (n + 10)(n - 7) = 0$, $n = 7$]

3. $142_n = 98_{10}$

ANSWER: n = 8
[$n^2 + 4n + 2 = 98$, $n^2 + 4n - 96 = (n + 12)(n - 8) = 0$, $n = 8$]

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1. Find dy/dx given $2x^2 - xy + 2y^2 = 10$

ANSWER: dy/dx = (4x – y)/(x – 4y) or (y – 4x)/(4y - x)
[4x – y – x(dy/dx) + 4y(dy/dx) = 0, 4x – y = (x – 4y)(dy/dx), dy/dx = (4x – y)/(x - 4y)]

2. Triangle ABC has sides a = 5, b = 7, c = 8. Find the value of cosA

ANSWER: 11/14
[cosA = $(b^2 + c^2 - a^2)/2bc$ = (49 + 64 – 25)/2(7)8 = 88/8(14) = 11/14]

3. Find the maximum value of the expression 12cosx – 8sinx

ANSWER: $4\sqrt{13}$
[$\sqrt{(144 + 64)} = \sqrt{208} = \sqrt{(16 \times 13)} = 4\sqrt{13}$

Find the domain of the given function

1. $f(x) = \sqrt{(x + 3)}/\sqrt{(3 - x)}$

ANSWER: {x: -3 ≤ x < 3}
[$(x + 3) \geq 0$, $x \geq -3$ and $(3 - x) > 0$, $3 > x$, $x < 3$, hence {x: $-3 \leq x < 3$}]

2. $g(x) = \sqrt{(x + 2)} / \sqrt[3]{(x - 2)}$

ANSWER: {x: x ≥ -2, x ≠ 2} [$\sqrt[3]{(x - 2)}$ is defined for all real x, but as a denominator, $x \neq 2$, $\sqrt{(x + 2)}$ is defined for $x \geq -2$, hence {x: $x \geq -2$, $x \neq 2$}]

3. $h(x) = \sqrt{(x - 1)}.\sqrt{(5 - x)}$

ANSWER: $\{x: 1 \le x \le 5\}$ [domain of $\sqrt{(x - 1)}$ is $x \ge 1$, and domain of $\sqrt{(5 - x)}$ is $x \le 5$, hence $\{x: 1 \le x \le 5\}$]

Find the inverse of the exponential function

1. $y = 5^{(x + 1)}$

ANSWER: $y = \log_5 x - 1$, or $y = \log_5(x/5)$
[$x = 5^{(y + 1)}$, $y + 1 = \log_5 x$, $y = \log_5 x - 1$, or $y = \log_5(x/5)$]

2. $y = 3^{-x}$

ANSWER: $y = - \log_3 x$, or $y = \log_3(1/x)$
[$x = 3^{-y}$, $- y = \log_3 x$, $y = - \log_3 x$ or $y = \log_3(1/x)$]

3. $y = 4^{2x}$

ANSWER: $y = ½\log_4 x$, or $y = \log_4\sqrt{x}$
[$x = 4^{2y}$, $2y = \log_4 x$, $y = ½\log_4 x = \log_4\sqrt{x}$]

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1. Find the coordinates of the turning points of the curve $y = 2x^3 - 3x^2 + 5$

ANSWER: (0, 5), (1, 4)
[$dy/dx = 6x^2 - 6x = 6x(x - 1) = 0$, $x = 0, 1$, for $x = 0$, $y = 5$, (0, 5),
for $x = 1$, $y = 2 - 3 + 5 = 4$, (1, 4)]

2. Solve the equation $\sin^2x - \cos^2x = 1$ for $0 < x < \pi$

ANSWER: $x = \pi/2$
[$-\cos2x = 1$, $\cos2x = -1$, $2x = \pi$, $x = \pi/2$]

3. Evaluate $(\log_2 36)(\log_2 125)/(\log_2 25)(\log_2 216)$

ANSWER: 1
[$(2\log_2 6)(3\log_2 5)/(2\log_2 5)(3\log_2 6) = 2(3)/ 2(3) = 6/6 = 1$]

Given that sinx = ½ and x is obtuse, evaluate and simplify

1. $1/(1 + \cos x)$

ANSWER: $4 + 2\sqrt{3}$

[$\sin x = ½$, $x = 150$, $\cos150 = -\cos30 = - \sqrt{3}/2$, $1/(1 + \cos x) = 1/(1 - \sqrt{3}/2) =$
$2/(2 - \sqrt{3}) = 2(2 + \sqrt{3}) = 4 + 2\sqrt{3}$]

2. $1/(1 + \tan x)$

ANSWER: $(3 + \sqrt{3})/2$

[sinx = ½, x = 150, tan150 = tan(-30) = - tan30 = -1/√3, 1/(1 + tanx) = 1/(1 – 1/√3) = √3/(√3 – 1) = √3(√3 + 1)/2 = (3 + √3)/2]

3. 1/(cosx + sinx)

ANSWER: -(1 + √3)

[sinx = ½, x = 150, cos150 = - √3/2, 1/(sinx + cosx) = 1/(1/2 - √3/2) = 2/(1 - √3) = 2(1 + √3)/-2 = -(1 + √3)]

Find the degree measures of the interior angles of a pentagon if

1. the exterior angles are in the ratio 2 : 3 : 3 : 5 : 5

ANSWER: 140°, 120°, 120°, 80°, 80°
[2x + 3x + 3x + 5x + 5x = 18x = 360, x = 20, exterior angles are 40, 60, 60, 100, 100, interior angles are 140, 120, 120, 80, 80]

2. the exterior angles are in the ratio 4 : 5 : 5 : 8 : 8

ANSWER: 132°, 120°, 120°, 84°, 84°
[4x + 5x + 5x + 8x + 8x = 30x = 360, x = 12, exterior angles are 48, 60, 60, 96, 96, interior angles are 132, 120, 120, 84, 84]

3. the exterior angles are in the ratio 1 : 2 : 3 : 4 : 5

ANSWER: 156°, 132°, 108°, 84°, 60°
[x + 2x + 3x +4x + 5x = 15x = 360, x = 24, exterior angles are 24, 48, 72, 96, 120, interior angles are 156, 132, 108, 84, 60]

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1. Find the inverse of the function f(x) = (3x + 2)/(2x – 3)

ANSWER: $f^{-1}(x)$ = (3x + 2)/(2x – 3)
[y = (3x + 2)/(2x – 3), x = (3y + 2)/(2y – 3), x(2y – 3) = 3y + 2, y(2x – 3) = 3x + 2
y = (3x + 2)/(2x – 3) = $f^{-1}(x)$ = (3x + 2)/(2x – 3)]

2. If 2 and 3 are the roots of the equation $x^2 + bx + c = 0$, evaluate $b^2 + c^2$.

ANSWER: 61
[b = -(2 + 3) = - 5, c = 2(3) = 6, $b^2 + c^2$ = 25 + 36 = 61]

3. Solve for x from the equation $(2/3)^x = (27/8)^{4/3}$

ANSWER: x = -4
[$(2/3)^x = (3/2)^4 = (2/3)^{-4}$, x = -4]

Factorise completely the cubic expression

1. $8x^3 + 64$

ANSWER: $8(x + 2)(x^2 – 2x + 4)$
[$8(x^3 + 8) = 8(x + 2)(x^2 – 2x + 4)$]

2. $250x^3 – 54y^3$

ANSWER: $2(5x – 3y)(25x^2 + 15xy + 9y^2)$
[$2(125x^3 – 27y^3) = 2((5x)^3 – (3y)^3) = 2(5x – 3y)(25x^2 + 15xy + 9y^2)$]

3. $64x^3 + 27y^3$

ANSWER: $(4x + 3y)(16x^2 - 12xy + 9y^2)$
[$(4x)^3 + (3y)^3 = (4x + 3y)(16x^2 - 12xy + 9y^2)$]

Two events A and B are such that $P(A) = 0.5$ and $P(B) = 0.8$. Find

1. $P(A∪B)$ if A and B are independent,

ANSWER: 0.9
[$P(A∩B) = 0.5(0.8) = 0.4$, $P(A∪B) = 0.5 + 0.8 – 0.4 = 0.9$]

2. $P(A∩B)$ if $P(A∪B) = 0.95$,

ANSWER: 0.45
[$P(A∩B) = 0.5 + 0.8 – 0.95 = 1.4 – 0.95 = 0.45$]

3. $P(A∪B)$ if $P(A∩B) = 0.48$

ANSWER: 0.82
$P(A∪B) = P(A) + P(B) - P(A∩B) = 0.5+ 0.8 – 0.48 = 0.82$]

---

1. The sum S_n of a sequence is given by $S_n = n^2 - 3n + 2$. Find the fourth term of the sequence.

ANSWER: $U_4 = 4$

[$U_4 = S_4 – S_3 = (16 – 12 + 2) – (9 – 9 + 2) = 6 – 2 = 4$]

2. If $y = (x^2 + 2x)^5$, find dy/dx

ANSWER: $dy/dx = 10(x + 1)(x^2 + 2x)^4$
[$dy/dx = 5(2x + 2)(x^2 + 2x)^4 = 10(x + 1)(x^2 + 2x)^4$

3. If the magnitude of the vector **a = 5i + xj** is 13, find the value of x.

ANSWER: $x = \pm 12$
[$25 + x^2 = 169$, $x^2 = 169 – 25 = 144$, $x = \pm12$]

Solve the equation for x. You may leave answer as a logarithm

1. $2(4^{2x}) + 4^x - 10 = 0$

ANSWER: $x = \log_4 2$, or log2/log4 or 1/2
[$2(4^x)^2 + 4^x - 10 = 0$. Let $y = 4^x$, $2y^2 + y - 10 = (2y + 5)(y - 2) = 0$, $4^x = 2$, $x = \log_4 2$, or x = log2/log4 = ½]

2. $6(5^{2x}) + 20(5^x) + 14 = 0$

ANSWER: No real solution
[$6(5^x)^2 + 20(5^x) + 14 = 0$. Let $y = 5^x$, $6y^2 + 20y + 14 = (6y + 14)(y + 1) = 0$,
$5^x = -1, -7/3$, no solution since 5^x is non-negative]

3. $3(7^{2x}) - 7^x - 4 = 0$

ANSWER: $x = \log_7(4/3)$ or x = log(4/3)/log7
[$3(7^x)^2 - 7^x - 4 = 0$. Let $y = 7^x$, $3y^2 - y - 4 = (3y - 4)(y + 1) = 0$, $7^x = 4/3$, $x = \log_7(4/3)$ or x = log(4/3)/log7]

Solve for x in the interval $0 < x < 90°$ the trigonometric equation:

1. $\sin 56° \cos x + \cos 56° \sin x = \sqrt{3}/2$

ANSWER: $x = 4°, 64°$
[$\sin(56 + x) = \sqrt{3}/2$, $56 + x = 60, 120$, $56 + x = 60$, $x = 4$, $56 + x = 120$, $x = 120 - 56 = 64$]

2. $\cos 148° \cos x + \sin 148° \sin x = ½$

ANSWER: $x = 88°$
[$\cos(148 - x) = ½$, $148 - x = 60$, $x = 148 - 60 = 88$]

3. $\sin 75° \cos x - \cos 75° \sin x = 1/\sqrt{2}$

ANSWER: $x = 30°$
[$\sin(75 - x) = 45, 135$, $75 - x = 45$, $x = 30$, $75 - x = 135$, $x = 75 - 135 < 0$]

---

1. If the term in x^2 in the expansion of $(1 + 2x)^4$ and that of $(1 + ax)^5$ are the same, find the value of a^2.

ANSWER: $a^2 = 12/5$
[$6(2x)^2 = 10(ax)^2$, $24 = 10a^2$, $a^2 = 24/10 = 12/5$]

2. Find the equation of a line with slope 3/4 and passing through the point A(3, 2).

ANSWER: **y = (3x – 1)/4**, or **y = (¾)x – 1/4** , or **3x – 4y = 1**
[(y - 2) = (3/4)(x – 3), y = (3x – 1)/4, or 3x – 4y = 1]

3. The first 2 terms of an exponential sequence are $2 - \sqrt{3}$ and $2 + \sqrt{3}$ respectively. Find the common ratio r. Simplify your answer.

ANSWER: $r = 7 + 4\sqrt{3}$
[$r = (2 + \sqrt{3})/(2 - \sqrt{3}) = (2 + \sqrt{3})^2 = (4 + 3) + 2(2)\sqrt{3} = 7 + 4\sqrt{3}$]

Find the amount needed for each concentration

1. A chemist needs to mix a 5% acid solution with a 10% acid solution to obtain 50 liters of an 8% acid solution.

ANSWER: 20 liters of 5% acid solution, 30 liters of 10% acid solution
[$0.05x + (50 - x)0.1 = 50(0.08)$, $5x + 10(50 - x) = 400$, $100 = 5x$, $x = 20$]

2. A chemist needs to mix a 30% acid solution with a 10% acid solution to obtain 50 liters of a 20% acid solution.

ANSWER: 25 liters of 30% acid solution, 25 liters of 10% acid solution
[$0.3x + (50 - x)0.1 = 50(0.2)$, $30x + 10(50 - x) = 1000$, $20x = 500$, $x = 25$]

3. A lab technician mixes a 10% alcohol solution with a 25% alcohol solution to obtain 30 liters of a 20% alcohol solution.

ANSWER: 10 liters of 10% alcohol solution, 20 liters of 25% alcohol solution
[$0.1x + (30 - x)(0.25) = 30(0.2)$, $10x + (30 - x)25 = 600$, $150 = 15x$, $x = 10$]

Solve the trigonometric equation for the interval $0 < x < \pi$.

1. $\tan^2 x = 1/3$

ANSWER: $x = \pi/6, 5\pi/6$ radians
[$\tan x = \pm 1/\sqrt{3}$, $\tan x = 1/\sqrt{3}$, $x = \pi/6$, $\tan x = -1/\sqrt{3}$, $x = (\pi - \pi/6) = 5\pi/6$]

2. $2\sin^2 x - \sqrt{2}\sin x = 0$

ANSWER: $x = \pi/4, 3\pi/4$ radians
[$2\sin x(\sin x - 1/\sqrt{2}) = 0$, $\sin x = 0$, no solution, $\sin x = 1/\sqrt{2}$, $x = \pi/4, 3\pi/4$]

3. $\cos^2 x = 1/4$

ANSWER: $x = \pi/3, 2\pi/3$ radians
[$\cos x = \pm 1/2$, $\cos x = 1/2$, $x = \pi/3$, $\cos x = -1/2$, $x = (\pi - \pi/3) = 2\pi/3$]

Find a relation between x and y given that

1. $x = t^2$, $y = 2t^3$

ANSWER: $y^2 = 4x^3$
[$x^3 = t^6$, $y^2 = 4t^6$, $y^2/x^3 = 4t^6/t^6$, $y^2 = 4x^3$]

2. $x = 2t - 1, y = 4t^2 + 2t$

ANSWER: $\mathbf{y = (x + 1)^2 + (x + 1)}$, or $\mathbf{y = x^2 + 3x + 2}$
$[2t = (x + 1), y = (x + 1)^2 + (x + 1) = x^2 + 2x + 1 + x + 1 = x^2 + 3x + 2]$

3. $x = 1/t^2, y = 2t$

ANSWER: $xy^2 = 4$, or $y^2 = 4/x$
$[t = (y/2), x = 1/(y/2)^2 = 4/y^2, xy^2 = 4$, or $y^2 = 4/x]$

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1. Find the equation of the perpendicular bisector of the line segment joining the origin and the point A(2, -1).

ANSWER: $4x - 2y - 5 = 0$, or $y = 2x - 5/2$
$[x^2 + y^2 = (x - 2)^2 + (y + 1)^2, 0 = -4x + 4 + 2y + 1, 2y - 4x + 5 = 0$ or $y = 2x - 5/2]$

2. Find all real solutions of the equation $x^5 + 64x^2 = 0$

ANSWER: $x = 0, -4$
$[x^2(x^3 + 64) = 0, x = 0$ or $x^3 = -64, x = -4]$

3. Find the solution set of the inequality $(x - 2)/(x + 2) < 0$

ANSWER: $\{x: -2 < x < 2\}$
$[(x - 2)(x + 2) < 0, -2 < x < 2]$

Express the given number as the difference of two squares of positive integers

1. 45

ANSWER: $7^2 - 2^2$ (49 – 4), $9^2 - 6^2$ (81 – 36), $23^2 - 22^2$ (529 – 484)

$[a^2 - b^2 = (a + b)(a - b) = 5 \times 9, a + b = 9, a - b = 5, 2a = 14, a = 7, b = 2]$
Alt. $a + b = 15, a - b = 3, 2a = 18, a = 9, b = 6, a + b = 45, a - b = 1, 2a = 46, a = 23, b = 22]$ (NB: Any one of the differences is sufficient)

2. 32

ANSWER: $6^2 - 2^2$ (36 – 4), $9^2 - 7^2$ (81 – 49)

$[a^2 - b^2 = (a + b)(a - b) = 32, a + b = 8, a - b = 4, a = 6, b = 2,$
Alt. $a + b = 16, a - b = 2, a = 9, b = 7,]$

3. 15

ANSWER: $4^2 - 1^2$ (16 – 1), $8^2 - 7^2$ (64 – 49)
$[a^2 - b^2 = (a + b)(a - b) = 15 = 5 \times 3, a + b = 5, a - b = 3, a = 4, b = 1,$
Alt. $(a + b)(a - b) = 15 \times 1 = a + b = 15, a - b = 1, 2a = 16, a = 8, b = 7]$

Find the inverse of the given logarithmic function.

1. $y = \log_a(x - 2)$

ANSWER: $y = a^x + 2$
[$x = \log_a(y - 2)$, $y - 2 = a^x$, $y = a^x + 2$]

2. $y = \log_2(2x + 1)$

ANSWER: $y = (2^x - 1)/2$
[$x = \log_2(2y + 1)$, $2y + 1 = 2^x$, $y = (2^x - 1)/2$]

3. $y = 2\log_3(x - 3)$

ANSWER: $y = 3^{x/2} + 3$
[$x/2 = \log_3(y - 3)$, $y - 3 = 3^{x/2}$, $y = 3^{x/2} + 3$]

A particle is moving in a straight line from a point O to a point A with a constant acceleration of $3m/s^2$. The velocity at A is 30m/s and it takes 10 seconds from O to A. Find

1. the initial velocity at O

ANSWER: 0 m/s

[$v = u + at$, $u = v - at = 30 - 30 = 0$]

2. the distance from O to A

ANSWER: 150 m
[$v^2 = 2as$, $s = v^2/2a = 30^2/2(3) = 900/6 = 150$ m]

3. the initial velocity at O if the velocity at A is 48 m/s and it takes 12 seconds from O to A].

ANSWER: 12 m/s
[$v = u + at$, $48 = u + 3(12)$, $u = 48 - 36 = 12$ m/s]

---

1. Find the remainder R when $f(x) = 3x^3 - 4x^2 + 2x - 10$ is divided by $(x - 2)$

ANSWER: R = 2
[$R = f(2) = 24 - 16 + 4 - 10 = 28 - 26 = 2$

2. Find the constant term in the binomial expansion of $(x^3 + 1/x)^8$

ANSWER: 28
[$8C_r x^{3r}(x^{-1})^{8-r} = 8C_r x^{4r-8}$, $4r - 8 = 0$, $r = 2$, $8C_2 = 8 \times 7/2 = 28$]

3. Find the values of b and c if the inequality $x^2 + bx + c < 0$ has solution set

$\{x: -5 < x < 3\}$

ANSWER: b = 2, c = -15

[$(x - 3)(x + 5) = x^2 + 2x - 15 < 0$, $b = 2$, $c = -15$]

Find the area of a rhombus having an angle of 60° if

1. a side has length 10cm,

ANSWER: $50\sqrt{3}$ cm^2

[Area = $a^2\sin\theta = 100\sin 60 = 100\sqrt{3}/2 = 50\sqrt{3}$ cm^2]

2. the shorter diagonal has length 20 cm,

ANSWER: $200\sqrt{3}$ cm^2

[longer diagonal = 2x, $x = 10\tan 60 = 10\sqrt{3}$, Area = $\frac{1}{2} d_1d_2 = \frac{1}{2}(20)(20\sqrt{3}) = 200\sqrt{3}$ cm^2]

3. the longer diagonal has length 20 cm.

ANSWER: $200/\sqrt{3}$ cm^2 or $200\sqrt{3}/3$ cm^2

[shorter diagonal = 2x, $x = 10\tan 30 = 10/\sqrt{3}$, Area = $\frac{1}{2} d_1d_2 = \frac{1}{2}(20)(20/\sqrt{3}) = 200/\sqrt{3} = 200\sqrt{3}/3$cm^2]

Simplify the complex fraction

1. $(x/y - 1 - 6y/x)/(x/y + 4 + 4y/x)$

ANSWER: $(x - 3y)/(x + 2y)$

[$(x^2 - xy - 6y^2)/(x^2 + 4xy + 4y^2) = (x - 3y)(x + 2y)/(x + 2y)(x + 2y) = (x - 3y)/(x + 2y)$]

2. $(6/(x + 3) - 4/(x - 4))/(2/(x - 4) + 5/(x + 3))$

ANSWER: $(2x - 36)/(7x - 14)$ or $2(x - 18)/7(x - 2)$

[$(6(x - 4) - 4(x + 3))/(2(x + 3) + 5(x - 4)) = (2x - 36)/(7x - 14)$]

3. $(4/(x - 3) + 3/(x + 3))/(3x/(x^2 - 9))$

ANSWER: $(7x + 3)/3x$

[$(4(x + 3) + 3(x - 3))/3x = ((4x + 3x) + (12 - 9))/3x = (7x + 3)/3x$]

Find the values of the constants a and b given that a particle is in equilibrium under the action of the forces

1. $(3i + 5j)$N, $(-7i + 2j)$N and $(ai + bj)$N

ANSWER: a = 4, b = - 7 [$(3i + 5j) + (-7i + 2j) + (ai + bj) = 0$, $3 - 7 + a = 0$, $a = 4$, $5 + 2 + b = 0$, $b = -7$]

2. $(-5i + 7j)$N, $(9i - 8j)$N, $(ai + bj)$N

ANSWER: a = -4, b = 1 [$(-5i + 7j) + (9i - 8j) + (ai + bj) = 0$, $-5 + 9 + a = 0$, $a = -4$, $7 - 8 + b = 0$, $b = 1$]

3. $(-4i + 5j)N$, $(-2i - 8j)N$, $(ai + bj)N$

ANSWER: a = 6, b = 3 $[(-4i + 5j) + (-2i - 8j) + (ai + bj) = 0, -4 - 2 + a = 0, a = 6, 5 - 8 + b = 0, b = 3]$

---

1. The sides of a right-angle triangle are x, 2x – 1 and 2x + 1. Find the value of x.

ANSWER: x = 8
$[(2x + 1)^2 = (2x - 1)^2 + x^2, 4x^2 + 4x + 1 = 4x^2 - 4x + 1 + x^2, x^2 - 8x = 0, x = 8]$

2. Find the second derivative of $y = 2x^4 - 3x^3 + 15x$.

ANSWER: $d^2y/dx^2 = 24x^2 - 18x$
$[dy/dx = 8x^3 - 9x^2 + 15, d^2y/dx^2 = 24x^2 - 18x]$

3. Solve the equation $\log_7(3x + 7) = 2$

ANSWER: x = 14
$[3x + 7 = 7^2 = 49, 3x = 42, x = 14]$

Find the solution set of the cubic inequality.

1. $(x + 2)(x - 3)(x - 5) > 0$

ANSWER: $\{x: x > 5, \text{ or } -2 < x < 3\}$

2. $(x - 1)(x - 3)(x + 1) < 0$

ANSWER: $\{x: x < -1, \text{ or } 1 < x < 3\}$

3. $(x + 3)(x - 2)(x + 2) > 0$

ANSWER: $\{x: -3 < x < -2, \text{ or } x > 2\}$

In a triangle, find the length

1. of a side if the area is $80cm^2$ and the altitude to that side has length 10cm,

ANSWER: 16 cm
$[A = ½ bh, 80 = ½ (b)10, b = 160/10 = 16 \text{ cm}]$

2. of an altitude if the area is 120 cm^2 and the side to which the altitude is drawn has length 12 cm.

ANSWER: 20 cm
$[A = ½ bh, 120 = (1/2)(12)h, 6h = 120, h = 20 \text{ cm}]$

3. of a side if the area is $27cm^2$ and the side is 3 cm longer than its altitude

ANSWER: 9cm
$[(1/2) x(x - 3) = 27, x^2 - 3x - 54 = (x - 9)(x + 6) = 0, x = 9]$

Find the inverse of the linear transformation

1. T: $(x, y) \to (2x + 3y, x + 2y)$

ANSWER: T^{-1}: $(x, y) \to (2x - 3y, -x + 2y)$

[$A = \begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix}$, $\det A = 4 - 3 = 1$, $A^{-1} = \begin{pmatrix} 2 & -3 \\ -1 & 2 \end{pmatrix}$, T^{-1}: $(x, y) \to (2x - 3y, -x + 2y)$]

2. T: $(x, y) \to (x + 3y, x + 4y)$

ANSWER: T^{-1}: $(x, y) \to (4x - 3y, -x + y)$

[$A = \begin{pmatrix} 1 & 3 \\ 1 & 4 \end{pmatrix}$, $\det A = 4 - 3 = 1$, $A^{-1} = \begin{pmatrix} 4 & -3 \\ -1 & 1 \end{pmatrix}$, T^{-1}: $(x, y) \to (4x - 3y, -x + y)$]

3. T: $(x, y) \to (2x + y, x + y)$

ANSWER: T^{-1}: $(x, y) \to (x - y, -x + 2y)$

[$A = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$, $\det A = 2 - 1 = 1$, $A^{-1} = \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}$, T^{-1}: $(x, y) \to (x - y, -x + 2y)$]

---

1. Find the volume obtained by rotating the curve $y = \sqrt{x}$ about the x-axis on the interval $4 \le x \le 10$.

ANSWER: 42π cubic units

[Volume = $\pi \int_4^{10} y^2 dx = \pi \int_4^{10} x dx = \pi x^2/2 \Big|_4^{10} = \pi(100/2 - 16/2) = \pi(50 - 8) = 42\pi$]

2. Given that $2x + 1 \le f(x) \le 16 - x^2$ over an interval containing $x = 3$, find $\lim_{x \to 3} f(x)$.

ANSWER: 7

[$\lim_{x\to 3} (2x + 1) \le \lim_{x\to 3} f(x) \le \lim_{x\to 3} (16 - x^2)$, $7 \le \lim_{x\to 3} f(x) \le 7$, hence $\lim_{x\to 3} f(x) = 7$]

3. Find the values of the constants a and b given that the inequality $x^2 + ax + b > 0$ has solution set {x: x < -5, or x > 8}

ANSWER: a = -3, b = -40

[$(x + 5)(x - 8) > 0$, $x^2 - 3x - 40 > 0$, a = -3 and b = -40]

Find the area of a segment of a circle of radius 10 cm if the angle at the center is

1. 60°

ANSWER: $50(\pi/3 - \sqrt{3}/2)$ cm², or $25(2\pi - 3\sqrt{3})/3$ cm²

[Area = $\frac{1}{2} r^2(\theta - \sin\theta) = 50(\pi/3 - \sqrt{3}/2) = 25(2\pi - 3\sqrt{3})/3$]

2. 45°

ANSWER: **$50(\pi/4 - 1/\sqrt{2})cm^2$**, or **$25(\pi - 2\sqrt{2})/2cm^2$**]
[Area = $\frac{1}{2} r^2(\theta - \sin\theta) = 50(\pi/4 - 1/\sqrt{2}) = 25(\pi - 2\sqrt{2})/2$]

3. 120°

ANSWER: $50(2\pi/3 - \sqrt{3}/2)cm^2$, or $25(4\pi - 3\sqrt{3})/3cm^2$
[Area = $\frac{1}{2} r^2(\theta - \sin\theta) = 50(2\pi/3 - \sqrt{3}/2) = 25(4\pi - 3\sqrt{3})/3$]

Simplify the complex fraction

1. $(1 + 8/x + 12/x^2)/(1 + 6/x + 8/x^2)$

ANSWER: $(x + 6)/(x + 4)$ [$(x^2 + 8x + 12)/(x^2 + 6x + 8) = (x + 6)(x + 2)/(x + 4)(x + 2) = (x + 6)/(x + 4)$]

2. $(1 + 8/x^3 + 15/x^6)/(1 + 9/x^3 + 18/x^6)$

ANSWER: $(x^3 + 5)/(x^3 + 6)$ [$(x^6 + 8x^3 + 15)/(x^6 + 9x^3 + 18) = (x^3 + 3)(x^3 + 5)/(x^3 + 3)(x^3 + 6) =(x^3 + 5)/(x^3 + 6)$]

3. $(1 + 3/x^2 + 2/x^4)/(1 + 4/x^2 + 3/x^4)$

ANSWER: $(x^2 + 2)/(x^2 + 3)$ [$(x^4 + 3x^2 + 2)/(x^4 + 4x^2 + 3) = (x^2 + 1)(x^2 + 2)/(x^2 + 1)(x^2 + 3)= (x^2 + 2)/(x^2 + 3)$]

Find dy/dx from the implicit equation

1. $5x^2 - 2xy - 3y^2 = 5$

ANSWER: $dy/dx = (5x - y)/(x + 3y)$
[$10x - 2y - 2xdy/dx - 6ydy/dx = 0$, $dy/dx(2x + 6y) = 10x - 2y$,
$dy/dx = (5x - y)/(x + 3y)$]

2. $3x^2 - 3xy + 4y^2 = 10$

ANSWER: $dy/dx = (6x - 3y)/(3x - 8y)$
[$6x - 3y - 3xdy/dx + 8ydy/dx = 0$, $6x - 3y = (dy/dx)(3x - 8y)$,
$dy/dx = (6x - 3y)/(3x - 8y)$]

3. $2x^2 + 5xy - 4y^2 = 20$

ANSWER: $dy/dx = (4x + 5y)/(8y - 5x)$
[$4x + 5y + 5x(dy/dx) - 8y(dy/dx) = 0$, $(4x + 5y) = (dy/dx)(8y - 5x)$.
$dy/dx = (4x + 5y)/(8y - 5x)$]

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1. Solve the equation $\tan^2 x + \tan x = 0$ for $-90° < x < 90°$

ANSWER: x = 0°, -45°

[tanx(tanx + 1) = 0, tanx = 0, x = 0°, tanx = -1, x = -45°]

2. Find n given $121_n - 43_n = 33_{10}$

ANSWER: n = 7

[$n^2 + 2n + 1 - 4n - 3 = n^2 - 2n - 2 = 33$, $n^2 - 2n - 35 = (n-7)(n+5) = 0$, n = 7]

3. Find x given that the vectors **a** = 4**i** + x**j** and **b** = **i** – 4x**j** are perpendicular.

ANSWER: x = ± 1

[**a**.**b** = $4 - 4x^2 = 0$, $x^2 = 1$, x = ± 1]

Integrate the given expression with respect to x.

1. $(x^4 - x^2)/(x^2 + x)$

ANSWER: $x^3/3 - x^2/2 + C$

[$(x^2 + x)(x^2 - x)/(x^2 + x) = x^2 - x$, $\int (x^2 - x)dx = x^3/3 - x^2/2 + C$]

2. $(x^4 - 16)/(x^2 + 4)$

ANSWER: $x^3/3 - 4x + C$

[$(x^2 + 4)(x^2 - 4)/(x^2 + 4) = x^2 - 4$, $\int (x^2 - 4)dx = x^3/3 - 4x + C$]

3. $(x^3 - 1)/(x - 1)$

ANSWER: $x^3/3 + x^2/2 + x + C$

[$(x - 1)(x^2 + x + 1)/(x - 1) = (x^2 + x + 1)$, $\int (x^2 + x + 1)dx = x^3/3 + x^2/2 + x + C$]

A and B are acute angles such that sinA = 4/5 and sinB = 12/13. Find the value of

1. sin(A – B)

ANSWER: - 16/65

[sin(A – B) = sinAcosB – cosAsinB = (4/5)(5/13) – (3/5)(12/13) = (20 – 36)/65 = –16/65]

2. cos(A + B)

ANSWER: - 33/65

[cos(A + B) = cosAcosB – sinAsinB = (3/5)(5/13)- (4/5)(12/13) = (15 – 48)/65 = - 33/65]

3. $\cos(B - A)$

ANSWER: 63/65

[$\cos(B - A) = \cos B\cos A + \sin B\sin A = (5/13)(3/5) + (12/13)(4/5) =$ $(15 + 48)/65 = 63/65$]

A bag contains 8 white balls, 7 black balls and 5 red balls. Two balls are drawn at random one after the other from the bag without replacement. Find the probability

1. one ball is white and one black

ANSWER: 28/95

[$A = \{WB, BW\}$, $P(A) = (8/20)(7/19) + (7/20)(8/19) = 28/95$]

2. one ball is black and one red

ANSWER: 7/38

[$C = \{BR, RB\}$, $P(C) = (7/20)(5/19) + (5/20)(7/19) = 7/(2 \times 19) = 7/38$]

3. one ball is red and one white

ANSWER: 4/19

[$D = \{RW, WR\}$, $P(D) = (5/20)(8/19) + (8/20)(5/19) = 4/19$

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1. Find the zeros of the function $f(x) = 3x^3 - 2x^2 - 27x + 18 = 0$.

ANSWER: $x = \pm 3, 2/3$

[$x^2(3x - 2) - 9(3x - 2) = (x^2 - 9)(3x - 2) = 0$, $x = \pm 3, 2/3$]

2. Simplify $(3\sqrt{3} - 4\sqrt{2})(3\sqrt{3} + 4\sqrt{2})$

ANSWER: - 5

[$(3\sqrt{3})^2 - (4\sqrt{2})^2 = 27 - 32 = -5$]

3. If the sides of a right-angle triangle are x, 2x + 2 and 2x + 3, find x.

ANSWER: x = 5

[$(2x + 3)^2 = (2x + 2)^2 + x^2$, $4x^2 + 12x + 9 = 4x^2 + 8x + 4 + x^2$, $x^2 - 4x - 5 = (x - 5)(x + 1) = 0$, $x = 5$]

Find the slope of the tangent to the given curve at the point A on the curve.

1. $2x^2 - 3xy + y^2 = 3$ at the point A(2, 1).

ANSWER: 5/4

[$4x - 3y - 3x(dy/dx) + 2y(dy/dx) = 0$, $dy/dx = (4x - 3y)/(3x - 2y) = (8 - 3)/(6 - 2) = 5/4$]

2. $x^2 + xy + y^2 = 7$ at the point A(1, 2),

ANSWER: -4/5
[$2x + y + x(dy/dx) + 2y(dy/dx) = 0$,
$dy/dx = -(2x + y)/(x + 2y) = -4/5$]

3. $2x^2 - xy - y^2 = 10$ at the point A(2, -3).

ANSWER: -11/4
[$4x - y - x dy/dx - 2y(dy/dx) = 0$,
$(dy/dx) = (4x - y)/(x + 2y) = (8 + 3)/(2 - 6) = -11/4$]

Find the coordinates of the vertex of the given quadratic curve

1. $y = 1 + 10x - x^2$

ANSWER: (5, 26)
[$-x^2 + 10x + 1 = -(x^2 - 10x) + 1 = -(x - 5)^2 + 25 + 1$, vertex = (5, 26)]

2. $y = 5 - 12x - x^2$

ANSWER: (-6, 41)
[$-x^2 - 12x + 5 = -(x^2 + 12x + 36) + 5 + 36 = -(x + 6)^2 + 41$, vertex = (-6, 41)]

3. $y = -20 + 8x - x^2$

ANSWER: (4, -4)
[$-(x^2 - 8x + 16) + 16 - 20 = -(x - 4)^2 - 4$, vertex = (4, -4)

A committee of 4 is to be chosen from 4 men and 4 women. In how many ways can this be done if

1. any one can be chosen,

ANSWER: 70
[$8C_4 = 8 \times 7 \times 6 \times 5 / 4 \times 3 \times 2 \times 1 = 70$]

2. there are 2 men and 2 women on the committee,

ANSWER: 36
[$4C_2 \times 4C_2 = ((4 \times 3)/2) \times ((4 \times 3)/2) = 6 \times 6 = 36$]

3. there are 3 women and 1 man on the committee?

ANSWER: 16
[$4C_3 \times 4C_1 = 4 \times 4 = 16$]

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1. A ball is dropped from a height 30m above the ground. Find the speed with which it hits the ground. Take $g = 10\ m/s^2$ (Leave answer as a surd)

ANSWER: $10\sqrt{6}$ m/s
[$v^2 = u^2 + 2as = 0 + 2(10)30 = 600$, $v = 10\sqrt{6}$ m/s]

2. A binary operation $*$ is defined on the set of integers Z by $a * b = a + b - 3$. Find the identity e for the operation

ANSWER: e = 3
[$a * e = a + e - 3 = a$, $e - 3 = 0$, $e = 3$]

3. Find the inverse of the function $f(x) = 3/x + 5$, defined for $x \neq 0$

ANSWER : $f^{-1}(x) = 3/(x - 5)$, for $x \neq 5$
[$y = 3/x + 5$, $x = 3/y + 5$, $xy = 3 + 5y$, $y(x - 5) = 3$, $y = 3/(x - 5)$, $f^{-1}(x) = 3/(x - 5)$ for $x \neq 5$]

Find the first three terms of an exponential sequence if they are represented by

1. $(x + 4), x, (x - 6)$

ANSWER: -8, -12, - 18
[$x^2 = (x + 4)(x - 6) = x^2 - 2x - 24$, $2x + 24 = 0$, $x = -12$, hence - 8, - 12, - 18]

2. $(x - 4), x, (x + 6)$

ANSWER: 8, 12, 18
[$x^2 = (x - 4)(x + 6) = x^2 + 2x - 24$, $2x - 24 = 0$, $x = 12$, hence 8, 12, 18]

3. $(x + 4), x, (x - 2)$

ANSWER: 8, 4, 2
[$x^2 = (x + 4)(x - 2) = x^2 + 2x - 8$, $2x - 8 = 0$, $x = 4$, hence 8, 4, 2]

Express in the form of an inequality the set of points in the x - y plane

1. inside the circle with center (2, -3) and radius 5,

ANSWER: $\{(x, y): (x - 2)^2 + (y + 3)^2 < 25\}$

2. inside and on the circle with center (-3, 4) and radius 4,

ANSWER: $\{(x, y): (x + 3)^2 + (y - 4)^2 \leq 16\}$

3. outside the circle with center (4, -2) and radius 2,

ANSWER: $\{(x, y): (x - 4)^2 + (y + 2)^2 > 4\}$

Find the exact value of

1. $\sin^{-1}(\sin(5\pi/3))$

ANSWER: $-\pi/3$
[$\sin(5\pi/3) = \sin(2\pi - \pi/3) = \sin(-\pi/3)$, $\sin^{-1}(\sin(-\pi/3)) = -\pi/3$]

2. $\tan^{-1}(\tan(5\pi/4))$

ANSWER: $\pi/4$
[$\tan(5\pi/4) = \tan(\pi + \pi/4) = \tan(\pi/4)$, $\tan^{-1}(\tan\pi/4) = \pi/4$]

3. $\cos^{-1}(\cos 4\pi/3)$

ANSWER: $2\pi/3$
[$\cos(4\pi/3) = \cos(2\pi - 2\pi/3) = \cos(-2\pi/3) = \cos(2\pi/3)$, $\cos^{-1}(\cos 2\pi/3) = 2\pi/3$]

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1. Find the cosine of the angle θ between the vectors **a = 12i – 5j** and **b = 4i + 3j.**

ANSWER: $\cos\theta = 33/65$
[$\cos\theta = a.b/|a||b| = (48 – 15)/13(5) = 33/65$]

2. Find the quadratic equation with roots $\sqrt{5} \pm 2$.

ANSWER: $x^2 - 2\sqrt{5}x + 1 = 0$
[sum of roots = $2\sqrt{5}$, product = $(\sqrt{5} + 2)(\sqrt{5} – 2) = 5 – 4 = 1$, $x^2 - 2\sqrt{5}x + 1 = 0$]

3. Expand and simplify $(1 + \sqrt{3})^3$

ANSWER: $10 + 6\sqrt{3}$
[$1 + 3\sqrt{3} + 3(\sqrt{3})^2 + (\sqrt{3})^3 = (1 + 9) + 3\sqrt{3} + 3\sqrt{3} = 10 + 6\sqrt{3}$]

Solve for x from the radical equation

1. $2\sqrt{(3x – 2)} = x + 2$

ANSWER: $x = 6, 2$
[$4(3x – 2) = x^2 + 4x + 4$, $x^2 – 8x + 12 = (x – 6)(x – 2) = 0$, $x = 2, 6$]

2. $x - \sqrt{(4x – 3)} = 2$

ANSWER: $x = 7$
[$x – 2 = \sqrt{(4x – 3)}$, $x^2 – 4x + 4 = 4x – 3$, $x^2 - 8x + 7 = (x - 7)(x - 1) = 0$, $x = 7$]

3. $\sqrt{(x^2 + 3x – 3)} = 5$

ANSWER: $x = -7, 4$
[$x^2 + 3x – 3 = 25$, $x^2 + 3x – 28 = (x + 7)(x – 4) = 0$, $x = -7, 4$]

Given that $0° < x < 180°$, solve the trigonometric equation

1. $\tan x = -\tan 36°$

ANSWER: $x = 144°$
[$\tan x = -\tan 36 = \tan(180 - 36)$, $x = 180 - 36 = 144$]

2. $\tan x = -\tan 100°$

ANSWER: $x = 80°$
[$\tan x = -\tan 100 = \tan(180 - 100)$, $x = 180 - 100 = 80$]

3. $\tan x = -\tan 123°$

ANSWER: $x = 57°$
[$\tan x = -\tan 123 = \tan(180 - 123)$, $x = 180 - 123 = 57$]

A linear transformation is defined by $T: (x, y) \to (x - 2y, -3x + 5y)$. Find the values of x and y given that the image of the point $A(x, y)$ is the point

1. $(1, -2)$

ANSWER: $x = -1, y = -1$
[$x - 2y = 1$, $-3x + 5y = -2$, $3(x - 2y) + (-3x + 5y) = 3 - 2 = 1$, $y = -1$, $x = -1$]

2. $(2, 3)$

ANSWER: $x = -16, y = -9$
[$x - 2y = 2$, $-3x + 5y = 3$, $3(x - 2y) + (-3x + 5y) = 9$, $-y = 9$, $y = -9$, $x = -16$]

3. $(-3, 5)$

ANSWER: $x = 5, y = 4$
[$x - 2y = -3$, $-3x + 5y = 5$, $3(x - 2y) + (-3x + 5y) = 3(-3) + 5 = -4$,
$y = 4$, $x = 5$]

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1. Evaluate and simplify $(\cos 30° + \sin 60°)/(\tan 45° - \tan 60°)$

ANSWER: $-(3 + \sqrt{3})/2$
[$(\sqrt{3}/2 + \sqrt{3}/2)/(1 - \sqrt{3}) = \sqrt{3}/(1 - \sqrt{3}) = \sqrt{3}(1 + \sqrt{3})/(1 - 3) = -(3 + \sqrt{3})/2$]

2. Given $3x^2 - 2xy + y^2 = 6$ evaluate dy/dx at $(1, -1)$

ANSWER: 2
[$6x - 2y - 2x dy/dx + 2y dy/dx = 0$, $(dy/dx)(2x - 2y) = 6x - 2y$, $dy/dx = (3x - y)/(x - y) = 4/2 = 2$]

3. Find the integral $\int \frac{(2x^3 - 3x^2 + 5)}{x^2} dx$

ANSWER: $x^2 - 3x - 5/x + C$

$[\int (2x - 3 + 5/x^2)dx = x^2 - 3x - 5/x + C]$

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1. The sum of three consecutive even integers is at least 24 and at most 36. List all possible values for the 3 integers.

ANSWER: {6, 8, 10}, {8, 10, 12}, {10, 12, 14} (1 mark for each triple)

$[24 \le (x - 2) + x + (x + 2) \le 36, 24 \le 3x \le 36, 8 \le x \le 12, x = 8, 10, 12]$

2. The sum of three consecutive odd integers is at least 39 and at most 51. List all possible values for the 3 integers.

ANSWER: {11, 13, 15} , {13, 15, 17}, {15, 17, 19}

$[39 \le (x - 2) + x + (x + 2) \le 51, 39 \le 3x \le 51, 13 \le x \le 17, x = 13, 15, 17]$

3. The sum of three consecutive integers is at least 33 and at most 39. List all possible values for the 3 integers,

ANSWER: {10, 11, 12}, {11, 12, 13}, {12, 13, 14}

$[33 \le (x - 1) + x + (x + 1) \le 39, 33 \le 3x \le 39, 11 \le x \le 13, x = 11, 12, 13]$

Factorize the cubic polynomial completely.

1. $x^3 + x^2 - 10x + 8$

ANSWER: $(x - 1)(x - 2)(x + 4)$

$[(x - 1)$ is a factor, $x^3 + x^2 - 10x + 8 = (x - 1)(x^2 + ax - 8)$, $(a - 1)x^2 = 1x^2$, $a = 2$

$x^2 + 2x - 8 = (x + 4)(x - 2)$, hence $(x - 1)(x - 2)(x + 4)]$

2. $2x^3 - x^2 - 7x + 6$

ANSWER: $(x - 1)(x + 2)(2x - 3)$

$[(x - 1)$ is a factor, $2x^3 - x^2 - 7x + 6 = (x - 1)(2x^2 + ax - 6)$, $(a - 2)x^2 = -1x^2$, $a = 1$

$2x^2 + x - 6 = (2x - 3)(x + 2)$, hence $(x - 1)(x + 2)(2x - 3)]$

3. $x^3 - 2x^2 - 13x - 10$

ANSWER: $(x + 1)(x + 2)(x - 5)$

$[(x + 1)$ is a factor, $x^3 - 2x^2 - 13x - 10 = (x + 1)(x^2 + ax - 10)$, $(a + 1)x^2 = -2x^2$, $a = -3$

$x^2 - 3x - 10 = (x - 5)(x + 2)$, hence $(x + 1)(x + 2)(x - 5)]$

Find the quadratic equation with integer coefficients having as one of its roots

1. $3 + 2\sqrt{3}$

ANSWER: $x^2 - 6x - 3 = 0$
[roots are $3 \pm 2\sqrt{3}$, sum = 6, product = 9 – 12 = - 3, hence $x^2 - 6x - 3 = 0$]

2. $4 - 2\sqrt{2}$

ANSWER: $x^2 - 8x + 8 = 0$
[roots are $4 \pm 2\sqrt{2}$, sum = 8, product = 16 – 8 = 8, hence $x^2 - 8x + 8 = 0$]

3. $-5 + 2\sqrt{7}$

ANSWER: $x^2 + 10x - 3 = 0$ [roots are $-5 \pm 2\sqrt{7}$, sum = -10, product = 25 – 28 = -3, hence $x^2 + 10x - 3 = 0$]

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1. If $\sin x = 1/\sqrt{2}$ and $\cos x < 0$, find x in the interval $0 < x < 2\pi$

ANSWER: $x = 3\pi/4$
[$\sin x > 0$ and $\cos x < 0$, hence x is in second quadrant, $x = 3\pi/4$]

2. The sum of the first n terms of a series is $S_n = 5n^2 - 3n$ for $n \geq 1$. Find the first three terms of the series.

ANSWER: $U_1 = 2$, $U_2 = 12$, $U_3 = 22$
[$U_1 = S_1 = 5 - 3 = 2$, $U_2 = S_2 - S_1 = (20 - 6) - 2 = 14 - 2 = 12$, $U_3 = S_3 - S_2 = (45 - 9) - 14 = 45 - 23 = 22$]

3. In a circle, the radius is 10 cm and an arc subtends an angle of 60° at the center. Find the area of the sector formed by the arc and the relevant radii.

ANSWER: $50\pi/3$ cm²
[$60° \equiv \pi/3$ radians, $A = \frac{1}{2} r^2\theta = \frac{1}{2}(100)\pi/3 = 50\pi/3$ cm²]

Solve the trigonometric equation for x in the interval $90° < x < 180°$

1. $\sin x = \sin 37°$

ANSWER: x = 143°
[$\sin x = \sin(180 - 37)$, $x = 180 - 37 = 143°$]

2. $\cos x = -\cos 65°$

ANSWER: x = 115°
[$-\cos 65 = \cos(180 - 65)$, $\cos x = \cos(180 - 65)$, $x = 180 - 65$, $x = 115°$]

3. $\tan x = -\tan 52°$

ANSWER: $x = 128°$
[$\tan x = \tan(180 - 52)$, $x = 180 - 52 = 128°$]

Find the coordinates of the vertices of a triangle if the vertices lie on the lines with equations

1. $x + y = 2$, $x - y = 4$, $2x - y = 4$

ANSWER: (3, 1), (2, 0), (0, -4)
[$x + y = 2$, $x - y = 4$, $2x = 6$, $x = 3$, $y = -1$, $(3, -1)$, $x + y = 2$, $2x - y = 4$, $3x = 6$, $x = 2$, $y = 0$, $(2, 0)$, $x - y = 4$, $2x - y = 4$, $x = 0$, $y = -4$, $(0, -4)$]

2. $y = x$, $y = -x + 2$, $y = 2x - 4$

ANSWER: (1, 1), (4, 4), (2, 0)
[$y = x$, $y = -x + 2$, $2y = 2$, $y = 1$, $x = 1$, $(1, 1)$; $y = x$, $y = 2x - 4$, $x - 4 = 0$, $x = 4$, $y = 4$, $(4, 4)$; $y = -x + 2$, $y = 2x - 4$, $-x + 2 = 2x - 4$, $3x = 6$, $x = 2$, $y = 0$, $(2, 0)$]

3. $x + y = 6$, $2x + y = 6$, $y = 2x$

ANSWER: (2, 4), (3/2, 3), (0, 6)
[$x + y = 6$ and $y = 2x$, $3x = 6$, $x = 2$, $y = 4$, $(2, 4)$, $2x + y = 6$ and $y = 2x$,
$4x = 6$, $x = 3/2$, $y = 3$, $(3/2, 3)$, $x + y = 6$ and $2x + y = 6$, $x = 0$, $y = 6$, $(0, 6)$]

Find values of the constants a and b given that the polynomial

1. $f(x) = ax^3 + bx^2 + x - 4$ is exactly divisible by $(x - 1)$ and $(x + 1)$.

ANSWER: $a = -1$, $b = 4$
[$a + b + 1 - 4= 0$, $a + b = 3$, $-a + b - 1 - 4 = 0$, $-a + b = 5$, $2b = 8$, $b = 4$, $a = -1$]

2. $f(x) = x^3 + ax^2 + bx - a$ has $(x - 1)$ and $(x + 2)$ as factors.

ANSWER: $a = 2$, $b = -1$
[$1 + a + b - a = 0$, $b = -1$, $-8 + 4a + 2 - a = 0$, $3a - 6 = 0$, $a = 2$]

3. $f(x) = 2x^3 + ax^2 + bx - 3$ is divisible by $(x - 1)$ and $(x + 1)$.

ANSWER: $a = 3$, $b = -2$
[$2 + a + b - 3 = 0$, $a + b = 1$, $-2 + a - b - 3 = 0$, $a - b = 5$, $2a = 6$, $a = 3$, $b = -2$]

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1. Under a translation, the point (2, 5) maps into the point (3, 2). Find the image of the point (-2, 3) under the same translation.

ANSWER: (-1, 0)

[u = (3, 2) – (2, 5) = (1, - 3), (-2, 3) + (1, -3) = (-1, 0)]

2. A binary operation is defined on the set of real numbers excluding -1 by $a * b = a + b + ab$. Find the identity e.

ANSWER: $e = 0$

[$a * e = a, a + e + ae = a, e(1 + a) = 0, e = 0$ since $a \neq -1$]

3. State the converse of the statement 'a regular polygon is equilateral' and determine if the converse is true or not.

ANSWER: 'An equilateral polygon is regular'. Converse is false.

Find the gradient dy/dx from the implicit equatio

1. $x^3 - x^2y + y^3 = 7$

ANSWER: $dy/dx = (3x^2 - 2xy)/(x^2 - 3y^2)$

[$3x^2 - 2xy - x^2dy/dx + 3y^2dy/dx = 0, (dy/dx)(x^2 - 3y^2) = 3x^2 - 2xy,$
$dy/dx = (3x^2 - 2xy)/(x^2 - 3y^2)$]

2. $2x^3 + x^2y - 2y^3 = 15$

ANSWER: $dy/dx = (6x^2 + 2xy)/(6y^2 - x^2)$

[$6x^2 + 2xy + x^2(dy/dx) - 6y^2(dy/dx) = 0, (dy/dx)(6y^2 - x^2) = (6x^2 + 2xy),$
$dy/dx = (6x^2 + 2xy)/(6y^2 - x^2)$]

3. $3x^3 - 2x^2y + y^3 = 20$

ANSWER: $dy/dx = (9x^2 - 4xy)/(2x^2 - 3y^2)$

[$9x^2 - 4xy - 2x^2(dy/dx) + 3y^2 (dy/dx) = 0, (dy/dx)(2x^2 - 3y^2) = 9x^2 - 4xy$
$dy/dx = (9x^2 - 4xy)/(2x^2 - 3y^2)$]

Given that $\cos A = -1/\sqrt{2}$ and A is obtuse, and $\sin B = \sqrt{3}/2$ and B is acute, evaluate

1. $\sin(A - B)$

ANSWER: $(1 + \sqrt{3})/2\sqrt{2}$, or $(1 + \sqrt{3})\sqrt{2}/4$, or $(\sqrt{2} + \sqrt{6})/4$

[$\sin(A - B) = \sin A\cos B - \cos A\sin B = (1/\sqrt{2})(1/2) - (-1/\sqrt{2})(\sqrt{3}/2) =$
$(1 + \sqrt{3})/2\sqrt{2}$]

2. $\cos(A + B)$

ANSWER: $-(1 + \sqrt{3})/2\sqrt{2}$, or $-(1 + \sqrt{3})\sqrt{2}/4$, or $-(\sqrt{2} + \sqrt{6})/4$

[cos(A + B) = cosAcosB – sinAsinB = (-1/√2)(1/2) – (1/√2)(√3/2) = (-1 - √3)/2√2]

3. sin(A + B)

ANSWER: (1 - √3)/2√2, or (1 - √3)√2/4, or (√2 - √6)/4
[sin(A + B) = sinAcosB + cosAsinB = (1/√2)(1/2) + (-1/√2)(√3)/2 = (1 - √3)/2√2]

A bag contains 8 white balls, 7 black balls and 5 red balls. Three balls are drawn at random one after the other from the bag without replacement. Find the probability

1. two balls are red and one black,

ANSWER: 7/114
[A = {RRB, RBR, BRR}, P(A) = (5/20)(4/19)(7/18) + (5/20)(7/19)(4/18) + (7/20)(5/19)(4/18) = 3(7)/(19)(18) = 7/19(6) = 7/114]

2. two balls are white and one red,

ANSWER: 7/57
[C = {WWR, WRW, RWW},P(C) = (8/20)(7/19)(5/18)+(8/20)(5/19)(7/18) + (5/20)(8/19)(7/18) = 3(2)(7)/(19)(18) = 7/19(3) = 7/57]

3. two balls are black and one white.

ANSWER: 14/95
[D = {BBW, BWB, WBB}, P(D) = (7/20)(6/19)(8/18) + (7/20)(8/19)(6/18) + (8/20)(7/19)(6/18) = 3(7/5)(2/9)(6/19) = 14/95]

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1. Find the stationary point of y = (2x + 1)/(x – 2)

ANSWER: NO STATIONARY POINT
[dy/dx = (2(x – 2) – 1(2x + 1))/$(x – 2)^2$ = -5/$(x – 2)^2$ ≠ 0, no stationary point]

2. The 5th term of an exponential sequence is 80 and the 8th term is 640. Find the general term U_n.

ANSWER: $U_n = 5(2^{n-1})$
[$ar^4 = 80$, $ar^7 = 640$, $r^3 = 640/80 = 8$, r = 2, a = 5, $U_n = ar^{n-1} = 5(2^{n-1})$]

3. Two angles of a cyclic quadrilateral measure 53° and 115° respectively. Find the measures of the 2 remaining angles.

ANSWER: 127°, 65°
[supplement of 53° is 127°, supplement of 115° is 65°, hence 127°, 65°]

Find the values of the constants a and b such that the quadratic inequality has the given solution set.

1. $ax^2 + bx + 2 > 0$ has solution set $\{x: -1 < x < 2\}$

ANSWER: a = -1, b = 1
[$(x + 1)(x - 2) = x^2 - x - 2 < 0$, $-x^2 + x + 2 > 0$, a = -1, b = 1]

2. $ax^2 + bx + 6 < 0$ has solution set $\{x: x < -3, \text{ or } x > 2\}$

ANSWER: a = -1, b = -1
[$(x + 3)(x - 2) = x^2 + x - 6 > 0$, $-x^2 - x + 6 < 0$, a = -1, b = -1]

3. $ax^2 + bx + 2 > 0$ has solution set $\{x: -1/2 < x < 2\}$

ANSWER: a = -2, b = 3
[$(2x + 1)(x - 2) = 2x^2 - 3x - 2 < 0$, $-2x^2 + 3x + 2 > 0$, a = -2, b = 3]

Find the common ratio r of an exponential sequence whose first three terms are given by

1. ...(x + 3), (x – 1), (x – 3)

ANSWER: r = 1/2

[$(x - 1)^2 = (x + 3)(x - 3)$, $x^2 - 2x + 1 = x^2 - 9$, $2x = 10$, $x = 5$,
$r = (x - 1)/(x + 3) = (5 - 1)/(5 + 3) = 4/8 = ½$]

2. (x – 2), (x + 1), (x + 3)

ANSWER: r = 2/3
[$(x + 1)^2 = (x - 2)(x + 3)$, $x^2 + 2x + 1 = x^2 + x - 6$, $x = -7$,
$r = (x + 1)/(x - 2) = (-7 + 1)/(-7 - 2) = -6/-9 = 2/3$]

3. (x + 1), (x + 3), (x + 4)

ANSWER: r = 1/2
[$(x + 3)^2 = (x + 1)(x + 4)$, $x^2 + 6x + 9 = x^2 + 5x + 4$, $x = 4 - 9 = -5$,
$r = (x + 3)/(x + 1) = (-5 + 3)/(-5 + 1) = -2/-4 = 1/2$]

Find the amount invested at each rate of interest if a man invests

1. GHs60,000 partly at 9% and the remainder at 6% and receives a total interest of GHs4,800 at the end of the year,

ANSWER: GHs40,000 at 9%, GHs20,000 at 6%
[$0.09x + (60{,}000 - x)(0.06) = 4{,}800$, $9x + (60{,}000 - x)6 = 480{,}000$
$3x = 480{,}000 - 360{,}000 = 120{,}000$, $x = 40{,}000$, $60{,}000 - 40{,}000 = 20{,}000$]

2. GHs100,000 partly at10% and the remainder at 15% and receives a total interest of GHs11,500 at the end of the year

ANSWER: GHs 70,000 at 10%, GHs 30,000 at 15%
[$0.1x + (100{,}000 - x)0.15 = 11500$, $15000 - 0.05x = 11500$, $5x = 350{,}000$,
$x = 70{,}000$, $100{,}000 - 70{,}000 = 30{,}000$]

3. GHs40,000 partly at 12% and the remainder at 8% and receives a total interest of GHs4,000 at the end of the year.

ANSWER: GHs20,000 at 12%, GHs20,000 at 8%
[$0.12x + 0.08(40{,}000 - x) = 4000$, $12x + 8(40{,}000 - x) = 400{,}000$
$4x = 80{,}000$, $x = 20{,}000$, $40{,}000 - 20{,}000 = 20{,}000$]

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1. If a = (x + 1)/(2x – 1), express (2a + 1)/(a – 1) in terms of x.

ANSWER: (4x + 1)/(2 – x)

[(2(x + 1) + (2x – 1))/((x + 1) – (2x – 1)) = (4x + 1)/(2 – x)]

2. Find a relation between x and y given that 2logx – 3logy = 1

ANSWER: $x^2/y^3 = 10$, or **$x^2 = 10y^3$**, or **$y^3 = x^2/10$**

[$\log(x^2/y^3) = \log 10$, $x^2/y^3 = 10$, or $x^2 = 10y^3$ or $y^3 = x^2/10$]

3. Find the domain of the function $y = \sqrt{(3 - x)}$.

ANSWER: $\{x: x \leq 3\}$

[function defined for $(3 - x) \geq 0$, $3 \geq x$, or $x \leq 3$]
